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Title: Compensating the Non-linear Distortions of an OFDM Signal with Neural Networks

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Compensating the Non-linear Distortions of an OFDM Signal with Neural Networks

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Abstract: This paper presents a non-linear distortion compensator for OFDM (Orthogonal Frequency Division Multiplexing) systems. OFDM signals are sensitive to non-linear distortions and different methods are studied to limit them. In the proposed technique, the correction is done at the receiver level by a higher-order neural network. Simulations show that the neural network brings perceptible gains in a complete OFDM system. In this paper we first present the OFDM, and explain why the non-linearities are a problem with this kind of modulation. Then we explain how we chose the neural network architecture, and finally some simulation results are presented.

Introduction

Multicarrier modulation, and especially OFDM, is now widely used for high speed communications over frequency selective channels. Examples of use are DAB (Digital Audio Broadcasting), DVB-T (Digital Video Broadcasting on Terrestrial networks), HiperLAN/II and IEEE 802.11a (radio local area networks). An OFDM system uses several low-rate sub-carriers to transmit data and can be used in time dispersive channels, such as multipath channels, with good efficiency [1]. Unfortunately, as an OFDM signal is the sum of multiple sinusoidal waves, it has a high peak to average power ratio (PAPR). This means that it is very sensitive to the non-linearities of the high power amplifier (HPA) [2]. The first obvious solution is to use a very linear HPA, but this solution is expensive and consumes too much power for portable systems.

One of the methods proposed to solve this problem is to reduce the PAPR by using special coding techniques [3]. These methods usually select codewords that produce low PAPR OFDM signals, and can be combined with error detection and correction systems.

Another method is to distort the signal before the HPA to compensate for its non-linearity [4].

A third technique is to correct the non-linearity at the receiver, using a post-distortion compensator. Such an idea has been proposed in [5] and upgraded in [6], where the compensator tries different symbols, simulates the OFDM system, including the HPA, and decides which symbol has been most likely emitted. Another compensator, proposed here, uses a neural network to correct the non-linearity introduced by the HPA.

OFDM

The basic idea of OFDM is to transmit data on parallel QAM (Quadrature Amplitude Modulation) or QPSK (Quadrature Phase Shift Keying) modulated sub-carriers. Let N be the number of sub-carriers, $C_k, k = 0 \ldots N-1$ the N complex symbols to be transmitted simultaneously, and T_S the OFDM symbol duration. The complex envelope of the ODFM base band signal is:

(1) $$S(t) = \sum_{k=0}^{N-1} C_k e^{2i\pi k \frac{t}{T_S}}$$

The OFDM symbol can be easily generated using a IFFT algorithm, and the reception can be done with a FFT to recover the C_k symbols. The most interesting property of OFDM is that the channel equalisation can be done in the frequency domain, after the FFT, and is a simple multiplication of the C_k symbols.

HPA

Then main source of non-linearities in a OFDM system is the HPA. It is the device that amplifies the signal to transmit it on radio waves. A simple model for the non-linear HPA can be used [2]:

(2) $$S_0(t) = S(t) \cdot G(|S(t)|)$$

And the function G depends on the chosen model for the HPA. Usually the HPA is very close to linear if the input signal is low enough, but when it increases the amplifier distorts the signal, and eventually it saturates. A parameter called Input

Back Off (IBO) indicates how much the transmitted signal is distorted by the HPA. It is the mean-saturation power ratio:

$$IBO = \frac{\left\langle \left|S(t)\right|^2 \right\rangle}{A_0^2} \tag{3}$$

Where A_0 is the output saturation amplitude. The lower the IBO, the more the signal is distorted.

Proposed system

To compensate for the non-linearities at the receiver, the proposed system uses a neural network before the QAM/QPSK demodulator, as shown in Fig. 1.

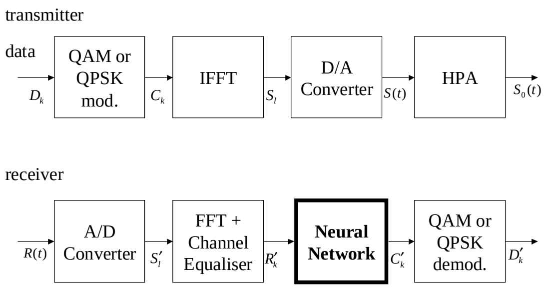

Fig. 1 Proposed system: the neural network corrects the received symbols

The main problem is that the non-linearity from the HPA is in the time domain, whereas the neural network is in the frequency domain. It can't be moved before the FFT because in this case it would have to do the channel equalising, which is much more complicated in the time domain. In the frequency domain the non-linearity is more complicated: intermodulations appears between the different carriers, so in fact each received symbol R'_k is a non-linear combination of the N transmitted symbols C_k.

The neural network has to reverse this combination: it must find back the C_k symbols, given the R'_k symbols.

Neural Network Architecture

We have shown [7] that the neural network doesn't have to learn the correction to apply to each carrier. If it can do the compensation for one carrier, it can be used to correct the other carriers, with a simple shift of its inputs. This means that we can divide the size of the output space by N, and thus have a simpler network, with only one complex output.

Several neural network architectures are adapted for multidimensional function approximation. The most popular are RBF and multilayer perceptrons [8]. The RBF network is not really adapted to this problem because the input data is scattered in all the dimensions, and not regrouped in a small number of regions: the C_k symbols aren't correlated, and all have uniform distributions. So a RBF would require approximately one prototype per possible OFDM symbol. As the number of different symbols rises exponentially with the number of carriers this is not a viable solution.

Multilayer perceptrons are more promising for this task. However the noticeable effect of the non-linearities in the frequency domain is intermodulation, which introduces higher-order disturbance on the carriers. That's why higher-order networks [9] have also been studied.

Indeed the networks that have shown the best performance for this task, both in terms of convergence and generalisation, are higher order networks, and especially the Ridge Polynomial Network (RPN) [10].

Given an input vector $x \in \mathbb{R}^d$, weight vectors $w_{ji} \in \mathbb{R}^d$, biases $b_{ji} \in \mathbb{R}$, and an activation function σ, the RPN's output is given by:

$$y = \sigma\left(\sum_{j=1}^{M}\prod_{i=1}^{j}\left(< x, w_{ji} > + b_{ji}\right)\right) \tag{4}$$

where $< a,b >$ is the inner product: $\sum_{i=1}^{d} a_i b_i$, and d is the input space dimension.

Each product term in equation (4) can be seen as the output of a j^{th} order pi-sigma network (described in [11]) with a linear activation function. M is the number of pi-sigma networks used, and the order of the RPN. Fig. 2 shows a diagram of the network architecture.

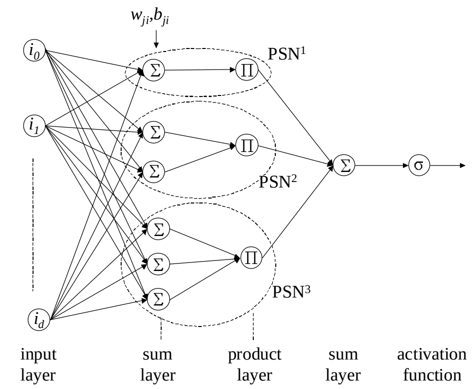

Fig. 2 RPN architecture. Only the first sum layer has adjustable weights. The neuron on the last layer is the only one to have a non-linear activation function. Each PSN^j is a jth order pi sigma network.

An incremental learning method can be used on the RPN. First, only the first-order PSN is trained. Then, the second-order PSN is added to the network, and trained. And so on, the other PSN are added and trained one by one [10]. Each PSN is trained using a method based on gradient descent [11].

The RPN is able to use the higher-order correlations of the input, and has fewer weights than the HPU (higher order unit) net, a more generic higher-order neural network. The presence of only one layer of adjustable weights results in a faster convergence of the gradient descent algorithm, compared to a multilayer perceptron [11].

Simulations and Results

This architecture has been tested successfully on a 4-carrier OFDM system. The OFDM system uses complex signals, whereas the neural network uses real signals. To adapt the neural network to the OFDM system, each complex signal has been separated into two real signals, the real and imaginary parts. Thus the input space dimension d of the neural network is $2N$, where N is the number of carriers, and two RPN are used, one to compute the real part of the output, and the other for the imaginary part.

The activation function used depends on the modulation used for the symbols C_k. With a QAM-16 modulation [12], each C_k can take 16 different values, and codes 4 bits of data. The real and imaginary parts can each take 4 values, A, $A/3$, $-A/3$ and $-A$, where A is the modulation amplitude. As a result, the neural network has to produce 4 different values on its outputs, so a special sigmoid function, with two intermediate plateaux is used as activation function, as shown in Fig. 3. Such activation functions have already been used in digital communications [13].

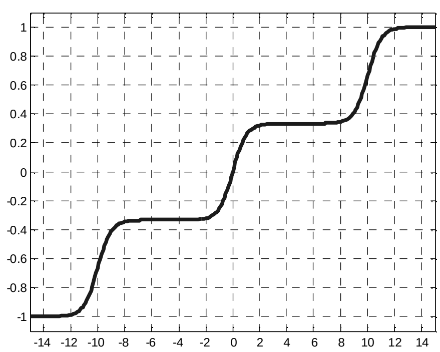

Fig. 3 Activation function used to adapt the network to a QAM-16 modulation. The targets of the neural network can take 4 different values.

To improve the convergence speed, a Levenberg-Marquardt algorithm [14] has been used instead of the standard gradient descent, and it proved to be very efficient. Each PSN needed less than 10 iterations to converge.

First a learning base is built with Cadence's Signal Processing Workshop (SPW) simulating an OFDM system with 4 carriers, a QAM 16 modulation and a channel with additive white gaussian noise (AWGN), with a signal to noise ratio E_b/N_0=13dB. The amplifier used has an IBO of 0dB, which means that the saturating power is equal to the mean power of the input signal. 8192 symbols are used as the learning base, and 8192 others as the validation set.

Then the RPN is trained with the learning base. The order M used is 7, as it seems to be the optimal order for this problem. Fig. 4 shows the evolution of the network Mean Square Error (MSE) during the learning process. Each spike comes from the introduction of a new pi-sigma network in the RPN. Each increase of the network order improves the performance, when the order is less than or equal to 7. When the order is greater than 7, the performance doesn't increase further. The training of each PSN is stopped when the magnitude of the weights update is less than a given threshold.

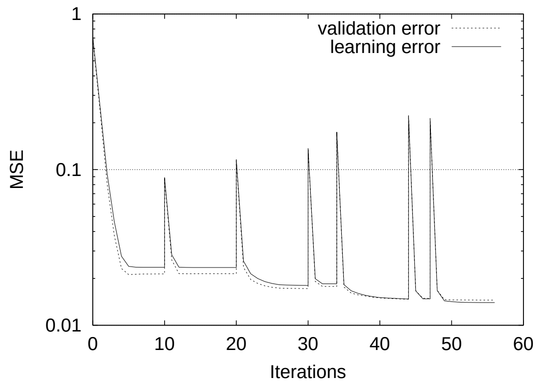

Fig. 4 Training of the RPN with a 4 carrier OFDM system, a QAM16 modulation, E_b/N_0=13dB, and IBO=0dB

Finally the neural network is simulated inside an OFDM system to determine its performance as a compensator. Fig. 5 shows the bit-error rate (BER) of the whole system with and without the neural network compensator. The curve for a system with an IBO of 1dB without neural network is also shown, for comparison. There are several manners to comment these results.

First we can study the gain without changing the amplifier or the signal power. With an IBO of 0dB and a signal to noise ratio E_b/N_0=16dB the neural network divides by 2 the bit error rate of the signal. This means that the error correcting code used in the digital communication has fewer errors to correct, and can be simpler, or permit a higher information bit rate.

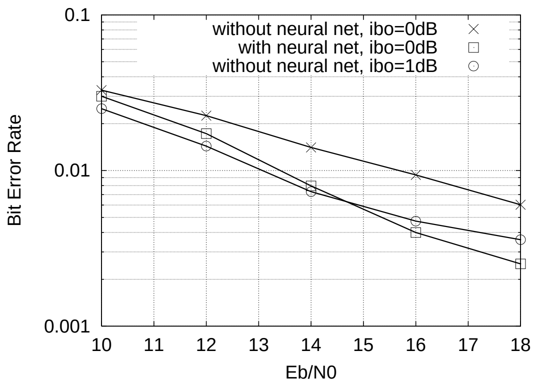

Fig. 5 Bit error rate of the OFDM system with and without the neural network. A QAM16 modulation is used on 4 carriers.

We can also want a fixed BER (10^{-2} as an example) and an IBO of 0dB. In that case the system without neural network requires a E_b/N_0=16dB, whereas the system with neural network requires only a E_b/N_0=13.5dB. This means that with the neural network we can divide the power of the signal and of the amplifier saturation by 1.8 (2.5dB) and still have the same performance. An amplifier with a lower saturation power is cheaper and consumes less power, so it is very interesting for a portable system.

Finally we can compare the curves of the system without neural network and IBO=1dB, and the one with the network and IBO=0dB. The two systems have similar results, and the higher the signal to noise ratio, the better the neural network gets. In fact the neural networks does as if the OFDM system had a higher quality amplifier. It manages to compensate some linear distortions introduced by the HPA.

These results are very promising and show that neural networks can be efficiently used in a OFDM system.

Conclusion

We have proposed a non-linear compensator for OFDM signals based on a neural network. The neural network is placed in the receiver, and corrects the non-linearities introduced by the transmitter’s high-power amplifier. The Ridge Polynomial Network showed good results in simulations and can improve the performance of OFDM systems, or keep the same performance with a lower power consumption. These results are very promising for this compensator, but the system currently only runs on 4-carrier modulations, and we carry on our research to adapt it to other systems with more carriers, closer to practical OFDM uses.

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